VOLUME 76 SEPARATE No. 16

ROCEEDINGS

AMERICAN SOCIETY OF CIVIL ENGINEERS

MAY, 1950

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By Guthlac Wilson, M. ASCE, and
G. M. J. Williams, Jun. ASCE

SOIL MECHANICS DIVISION

Headquarters of the Society
33 W. 39th St.
New York 18, N.Y.

PRICE $0.50 PER COPY

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

PAVEMENT BEARING CAPACITY COMPUTED BY THEORY OF LAYERED SYSTEMS

By Guthlac Wilson,[1] M. ASCE, and G. M. J. Williams,[2] Jun. ASCE

Synopsis

This paper describes a method for calculating the bearing capacity of rigid and flexible pavements, which is based on the theory of layered elastic systems. Graphs are presented from which the required pavement thickness can be obtained directly, if the loading and the physical properties of the pavement and subgrade materials are known. It is shown how these stresses can be allowed for in cohesive and in cohesionless soils.

Commonly used methods for computing the bearing capacity of pavements are discussed, particularly with reference to the degree in which they permit a count to be taken of the various factors affecting the bearing capacity of pavements.

Pavement thicknesses are calculated for typical cases by these methods, and also by the method described in the paper, and the results are compared.

1. Introduction

The main content of this paper is the application of the methods of the theory of elasticity to the calculation of the bearing capacity of pavements. No claim is made that this method is the best one for estimating the proper thickness of pavements. The writers believe, rather, that a semi-empirical method, such as the California bearing ratio (CBR) method,[3] is the soundest. They do believe, however, that it is advantageous to have a theoretical yardstick, applicable if the materials concerned were "perfect," with which to compare any empirical method. Such a yardstick should include all the factors that affect the bearing capacity of pavements. The method presented involves

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Note.—Written comments are invited for publication; the last discussion should be submitted by October 1, 1950.

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[3] "Development of CBR Flexible Pavement Design Method for Airfields: A Symposium," *Proceedings*, ASCE, January, 1949, p. 3.

the application of principles published by D. M. Burmister, Assoc. M. ASCE, in 1943[4] and again in 1945[5] under the descriptive name of "theory of layered systems." The writers believe that this method includes more relative factors than any other.

In 1946, discussing a paper by H. Q. Golder,[6] one of the writers (Mr. Wilson) made the suggestion that the bearing capacity of pavements should be calculated from stresses both in the pavement and in the soil as determined by the Burmister theory, rather than from the assumption of an arbitrary limiting deflection as proposed by Mr. Burmister. In 1946, L. Fox extended[7] Mr. Burmister's computations to the analysis of the stresses in the soil, thus opening the way for the determination of the bearing capacity of a pavement supported on a purely cohesive soil subjected to no initial shearing stresses.

In this paper the writers make allowance for the weight of the pavement and soil, and for the magnitude of the earth pressure at rest, and they extend the investigation to any elastic soil, the state of failure of which can be represented by the Coulomb equation:

$$\tau = c + \sigma \tan \phi \qquad \text{(1)}$$

in which τ is the intensity of shear stress; c is a factor denoting cohesion per unit area; σ is the intensity of direct stress; and ϕ is the angle of internal friction.

2. Factors Affecting the Bearing Capacity of Pavements

The methods of design which are most widely used at present are the method of H. M. Westergaard,[8] M. ASCE, for rigid pavements and the CBR method for flexible pavements.[9,10] The methods proposed by R. Glossop and Mr. Golder in 1944[11] and by Mr. Golder in 1946[6] (which are said to be applicable to rigid pavements) have had some use as have the methods proposed by the American Civil Aeronautics Authority[12] (CAA) and by Norman W. McLeod.[13] The former[12] is stated to be applicable to both rigid and flexible pavements and the latter[13] to flexible pavements only.

The factors affecting the bearing capacity of pavements, and the extent to which the various methods mentioned take account of these factors, are indicated in Table 1.

In all methods which involve the strength or elasticity of the foundation the uncertain factor is the deterioration of the foundation, and the difficulty is the decision as to how samples should be conditioned to make a sufficient, yet not excessive, allowance for this deterioration.

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[4] "The Theory of Stresses and Displacements in Layered Systems and Applications to the Design of Airport Runways," by D. M. Burmister, *Proceedings*, Highway Research Board, National Research Council, Vol. 23, 1943.

[5] "General Theory of Stresses and Displacements in Layered Systems," by D. M. Burmister, *Journal of Applied Physics*, Vol. 16, 1945, p. 89.

[6] "The Relationship of Runway Thickness and Undercarriage Design to the Properties of the Subsoil," by H. Q. Golder, *Airport Paper No. 4*, Inst. C. E., 1946.

[7] "Computation of Traffic Stresses in a Simple Road Structure," by L. Fox, *Proceedings*, 2d International Conference on Soil Mechanics, Rotterdam, 1948, Vol. II, p. 236.

[8] "Stresses in Concrete Pavements Computed by Theoretical Analysis," by H. M. Westergaard, *Public Roads*, April, 1926, pp. 25–35.

[9] "Soil Tests for the Design of Runway Pavements," by T. A. Middlebrooks and G. E. Bertram, *Proceedings*, Highway Research Board, National Research Council, Vol. 22, 1942, p. 144.

[10] "Preparation of Subgrades," by O. J. Porter, *ibid.*, Vol. 18, 1938, p. 324.

[11] "Construction of Pavements on a Clay Foundation Soil," by R. Glossop and H. Q. Golder, *Road Paper No. 15*, Inst. C. E., 1944.

[12] "Classification of Soils for Airport Construction," by Henry Aaron, *Proceedings*, 2d International Conference on Soil Mechanics, Rotterdam, 1948, Vol. V, p. 190.

[13] "An Investigation of Airport Runways in Canada," by Norman W. McLeod, *ibid.*, Vol. IV, p. 167.

A justifiable criticism of the CBR method is that, in some cases, tests on soaked specimens lead to lack of economy, as the foundation never attains as weak a condition during the life of the pavement, and many engineers now base their designs on the CBR of specimens conditioned to the weakest state which, in their judgment, the foundation will ever attain. Valuable work has been published on suitable methods of conditioning specimens[14] or of modifying the CBR found from soaked specimens.[15]

TABLE 1.—FACTORS TAKEN INTO ACCOUNT, IN VARIOUS DEGREE, BY VARIOUS METHODS OF PAVEMENT DESIGN

Abbreviations: A = Civil Aeronautics Authority; B = Burmister; C = California Bearing Ratio; G = Glossop and Golder (1944) and Golder (1946); M = McLeod; and W = Westergaard.

Factor	Directly	Indirectly	Possible	Not at all
Strength of pavement material	W, B	C, A, M	....	G
Elasticity of pavement material	W, B	C, A, M	....	G
Stresses in pavement except applied load[a]	....	C, A, M	W,[b] B[b]	G
Strength of foundation	G, C, M, B	A	....	W
Elasticity of foundation	W, B	C, A, M	....	G
Pre-existing stresses in foundation	B	C, A, M	G	W
Deterioration of foundation	C	A	B, W, G, M	....
Frequency of loading	....	C, A, M	B	W, G
Tire pressure	W, G, B	C, M	....	A

[a] Stresses from causes other than applied load, such as temperature change and shrinkage. [b] Can be allowed for by appropriate reduction in permissible tensile stress.

The CBR method applies only to the design of flexible pavements, but J. K. M. Henry and H. Grace[16] have applied it to the design of rigid pavements by assuming that a rigid pavement of a given thickness is equivalent to a flexible pavement of twice that thickness. Their experiment has been successful. Evidence has been presented[17] to show that this suggestion is compatible with other design methods.

Mr. McLeod's method[13] appears to result in a range of thicknesses generally proportional to, but thinner than, those derived by the CBR method and thus to have a lower factor of safety. The method adopted by the CAA, in which the soil properties are deduced from index tests on the materials, appears to lead to a lower factor of safety than does the CBR method in the case of weak materials and to a greater factor of safety in the case of strong materials.

3. PROPOSED BASIS OF COMPUTING BEARING CAPACITY

In 1943, Mr. Burmister[4] published a mathematical solution of the problem shown in Fig. 1, comprising a total load P uniformly distributed over a circle of radius a on the surface of a layer of elastic material, which in turn rests on a semi-infinite elastic body. The superficial layer of thickness, h, has elastic properties as follows: E_1 is Young's modulus of elasticity; and μ_1 is Poisson's ratio. The elastic constants for the underlying material are E_2 and μ_2, respectively.

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[14] "A Procedure for Evaluating the Influence of the Moisture Content of the Subgrade on the Thickness Required for Flexible Pavements of Airfields," by H. Loxton, H. M. Beavis, and M. D. McNicholl, *Proceedings*, 2d International Conference on Soil Mechanics, Rotterdam, 1948, Vol. II, p. 218.

[15] "Methods of Flexible Pavement Design: Wyoming Method of Flexible Pavement Design," by I. E. Russell and D. J. Olinger, *ibid.*, Vol. V, p. 165.

[16] "The Incorporation of Decomposed Granite in the Design and Construction of Pavements in Hong Kong," by J. K. M. Henry and H. Grace, *ibid.*, Vol. IV, p. 190.

[17] *Ibid.*, Vol. VI, p. 130.

Mr. Burmister has published formulas giving the stresses set up in both materials in terms of the two parameters, E_1/E_2 and h/a, assuming μ_1 and μ_2 to have the same value.[4] He considered both the case in which the materials are fully continuous with no relative movement at the interface, and the case which arises when the interface is frictionless so that no shear stress acts across it. Subsequently, Mr. Fox[7] published values for the stresses on the axis of symmetry in the lower material. These constitute an evaluation of the stresses set up in a simple pavement if it is assumed that the pavement material and the subgrade behave elastically, and that the load acts uniformly over a circular area. The writers have also assumed that the pavement and the subgrade are fully continuous at the interface.

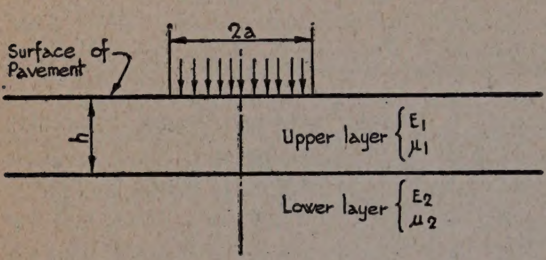

FIG. 1

Failure of such a pavement may begin in either of two ways: The pavement itself may fail initially, leading to overloading of the subgrade and to general failure; or the subgrade may fail first, causing the pavement to fail because it is inadequately supported. Consequently, the stresses in neither material can exceed the permissible stress for that material.

The pavement will nearly always begin to fail when the tensile stress at the base exceeds the permissible stress for the pavement material. This is particularly true of concrete pavements, and is the only method of pavement failure considered in this paper. The subgrade fails when the Coulomb equation (Eq. 1) is first fulfilled at any point on the axis of symmetry of the loaded area.

Mr. Burmister gives expressions for the tensile stress in the pavement on the axis of symmetry, from which Fig. 2 may be derived by numerical integration after the manner adopted by Mr. Fox. In Fig. 2, q represents the intensity of pressure on the loaded area; and σ_c, the resulting tensile stress in the base of the concrete pavement. Consequently, if σ_c is taken as the maximum tension allowable in the pavement, these curves will give the least thickness, h, necessary to prevent failure of the pavement in terms of a, the radius of the loaded area.

This allowance alone is not sufficient to insure a safe design because the strength of the subgrade has not been considered, and it may be necessary to increase the pavement thickness to prevent the subgrade from being overstressed. The stresses in the subgrade arise from two sources—the load applied on the surface and the dead weight of the soil and the pavement. The stresses due to the former are given by Mr. Fox; but, to estimate those set up through dead weight, a value must be assumed for the coefficient of subgrade earth pressure at rest, K_o. It will be shown subsequently in Section 5 that K_o tends to decrease with depth. The value to be used in the calculation is that which will apply at the critical depth. This depth is shown in the illustrative curves for the cases when $c = 0$, whereas in the cases when $\phi = 0$ the writers introduce a new parameter, χ, which allows for the variation of K_o with depth.

In this paper it is assumed that the subgrade and the pavement material have the same density, γ. If, then, any point on the axis of symmetry of the loaded area is considered, the stresses from these two causes can be added together to obtain the total resultant stresses at that point. Thus, the resultant vertical stress is given by

$$\sigma_{vr} = q\,\sigma_I + \sigma_v \quad \text{(2a)}$$

and the resultant horizontal stress by

$$\sigma_{hr} = q\,\sigma_{III} + \sigma_h \quad \text{(2b)}$$

in which σ_I and σ_{III} are, respectively, the vertical and horizontal stresses due to unit tire pressure, as derived by Mr. Fox; and σ_v and σ_h are the vertical and horizontal stresses pre-existing in the soil (see Section 5). Since the point in question is on the axis of symmetry, σ_{vr} and σ_{hr} will be the maximum and minimum principal stresses, so that failure will occur when Eq. 1 is satisfied:

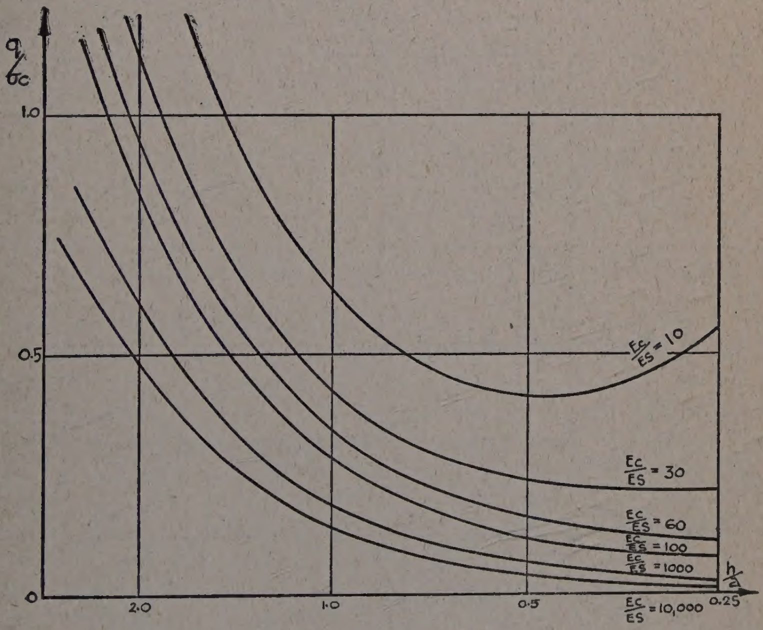

Fig. 2.—Permissible Values of q/σ_c If Concrete Failure Occurs Before Soil Failure

$$\frac{\sigma_{vr} - \sigma_{hr}}{2} = c + \frac{\sigma_{vr} + \sigma_{hr}}{2} \tan \phi \quad \text{(3a)}$$

or

$$\frac{q\,(\sigma_I - \sigma_{III}) + \sigma_v - \sigma_h}{2} = c + \left[\frac{q\,(\sigma_I + \sigma_{III}) + \sigma_v + \sigma_h}{2}\right] \tan\phi \quad ..(3b)$$

Eq. 3*b* gives the value of q that causes Eq. 1 to be satisfied at that point—that is, the value which causes failure to occur.

By repeating the process at several points on the axis of symmetry, the least value of q that involves failure anywhere can be found. This value wil give the maximum permissible tire pressure if failure is not to occur, and is shown by h/a-lines on a special set of design charts[17a] devised for various combinations of values for the parameters c, ϕ, E_1/E_2, and h/a. In this descriptive paper it is not feasible to reproduce a complete set of charts such as would be necessary in practice. The writers make use of fifteen such charts.[17a] In the first five a separate chart is drawn for E_1/E_2-ratios of 10, 100 (see Fig. 3), 300, 600, and 1,000, respectively. The values of other factors are: $\phi = 0$; $\chi = 0$ and 1; and $K_o = 1$ and maximum. Three sets of two each are for $K_o = 0.5$ (Fig. 4) and 0.75, respectively, with E_1/E_2-ratios at 10, 60, and 100.

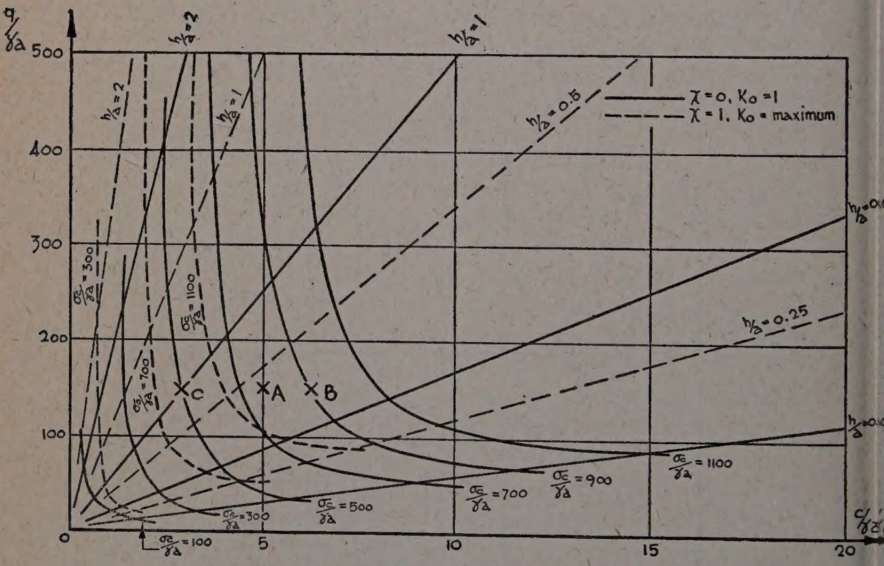

FIG. 3.—SAMPLE CHART $q/(\gamma\, a)$ VERSUS $c/(\gamma\, a)$; $\phi = 0$ AND $E_1/E_2 = 100$

Holding K_o constant at 0.5, two sets of two each are for $\phi = 30°$ ($E_1/E_2 = 10$ and 100) and $\phi = 37.5°$ ($E_1/E_2 = 10$ and 100). In these charts terms representing stresses or pressures have been divided by the term $\gamma\, a$ to obtain nondimensional results. Alternately for a fixed value of q these curves will give the minimum value of h/a if failure is not to occur in the subgrade, so that the least pavement thickness required to safeguard the subgrade can be found.

[17a] A complete set of full-scale drawings (order No. 16C) will be mailed postpaid on receipt of $2.00 (50% discount to ASCE members).

Thus, the theory gives two values for the pavement thickness: That derived from Fig. 2 which insures that the pavement itself is not overstressed, and that derived from the design charts such as Figs. 3 and 4 insuring that the subgrade is safe. To obtain a completely safe design it will be necessary to take the greatest of the several thicknesses thus determined.

For convenience the data in Fig. 2 are represented on the design charts by the $\sigma_c/(\gamma\ a)$-lines. These lines give the tire pressures that would cause the maximum permissible tensile stress to be set up in the base of the pavement. Several curves are given on each chart to allow for variations in the tensile strength of the pavement material.

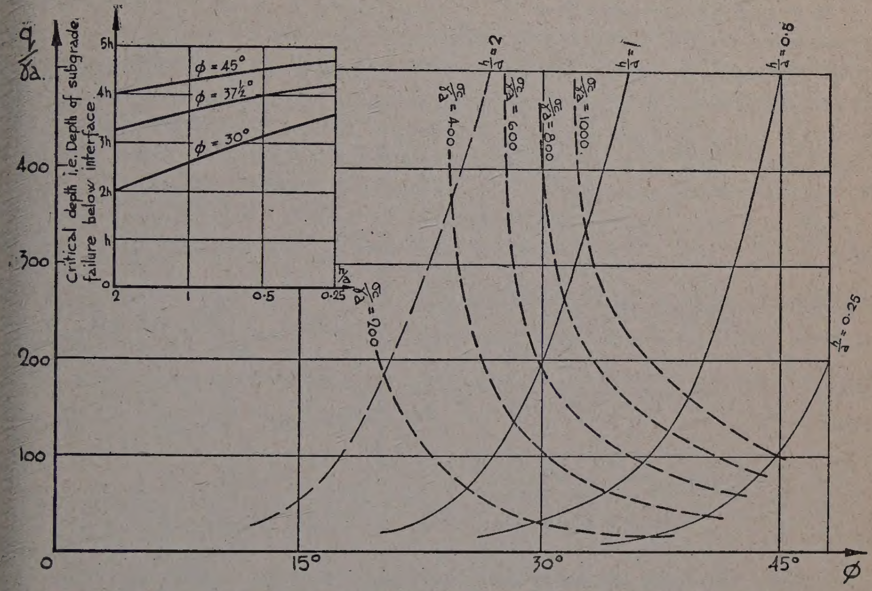

FIG. 4.—SAMPLE CHART $q/(\gamma\ a)$ VERSUS ϕ; $c = 0$, $E_1/E_2 = 60$, AND $K_o = 0.75$

An example of how the charts may be used to determine the pavement thickness necessary to meet a given set of circumstances is illustrated in Fig. 3. This particular chart is for a cohesive soil with no internal friction, for which the ratio $E_1/E_2 = 100$. There are two sets of curves relating to different values of K_o, which factor is fully discussed in Section 5. Consider the curves for the case when $K_o = 1$ and suppose $q/(\gamma\ a) = 150$ and $c/(\gamma\ a) = 5$ for a particular combination of subgrade and loading. These values, taken as ordinate and abscissa, respectively, fix point A in Fig. 3, corresponding approximately to $h = 0.70\ a$, which gives the least thickness required to safeguard the subgrade against failure. Suppose the pavement material is such that $\sigma_c/(\gamma\ a) = 900$. This value, in conjunction with $q/(\gamma\ a) = 150$ gives point B in Fig. 3, showing that the least thickness to prevent failure from occurring in the pavement is equal to $0.65\ a$.

The thickness that must be taken for design is the greater—namely, $0.70\ a$ corresponding to subgrade failure, which shows that the subgrade will fail before the pavement. On the other hand, if the pavement material were such that $c/(\gamma\ a) = 500$, point C, Fig. 3, would have been obtained, giving $h = 1.0\ a$ as the least thickness for safety. This thickness is larger than that necessary to safeguard the subgrade, so that now the pavement rules and the design thickness must be increased to 1.0 a.

4. The Physical Characteristics of the Soil

It is usually assumed that the state of stress of a soil at failure can be represented by Eq. 1 and that most soils fall into one of two classes—purely cohesive or cohesionless. The Mohr envelopes of these two classes are shown in full lines in Fig. 5.

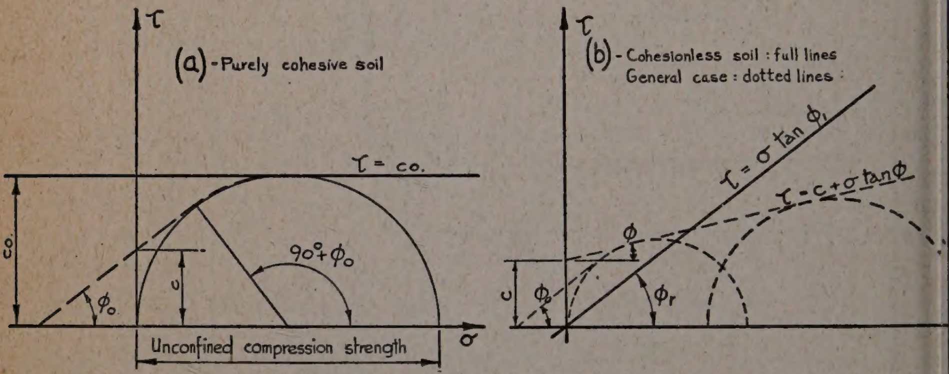
τ
(a) - Purely cohesive soil
τ = co.
co.
90°+φo
φo
c
Unconfined compression strength
σ
τ
(b) - Cohesionless soil : full lines
General case : dotted lines
τ = σ tan φr
τ = c + σ tan φ
φ
φr
c
φr

Fig. 5.—Mohr Envelopes of Two Classes of Soils

It does not seem probable that the tensile strength of a purely cohesive soil is equal to the compression strength and one of the writers (Mr. Wilson) has suggested[18] that a closer approximation to the true envelope of failure in such a case is as shown in heavy dotted lines in Fig. 5(a), the angle ϕ_o being that determined from the direction of the shear planes in an unconfined compression test specimen. In the general case of a soil having both cohesion and friction the envelope would be as shown in the dotted lines in Fig. 5(b). This type of envelope has been assumed in the preparation of the design charts typified by Figs. 3 and 4. The angle ϕ_o has been assumed to be 30° in cases when $\phi \leqq 30°$ and to be equal to ϕ, when $\phi > 30°$.

The envelope of failure and the modulus of elasticity should be determined on the basis of quick triaxial tests in the case of taxiways and warm-up areas. In the case of runways and roads it is possible that high speed tests (such as those at Harvard University[19] (Cambridge, Mass) on the investigation of the safety of the slopes of the Panama Canal) would be more suitable, in view of the very brief time that the load is being applied. The modulus of elasticity of soils is known to vary enormously with the minor principal stress.[20] Therefore, this stress should be suitably regulated during the tests, and an allowance should be made for the stresses existing in the soil before application of the wheel load. These stresses are considered in Section 5.

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[18] *Journal*, Inst. C. E., Vol. 24, 1945, p. 544.

[19] "Strength of Soils Under Dynamic Loads," by A. Casagrande and W. L. Shannon, in "Panama Canal—The Sea-Level Project: A Symposium," *Transactions*, ASCE, Vol. 114, 1949, p. 755.

[20] "A Contribution to the Study of the Elastic Properties of Sand," by G. Wilson and J. L. E. Sutton, *Proceedings*, 2d International Conference on Soil Mechanics, Rotterdam, 1948, Vol. I, p. 197.

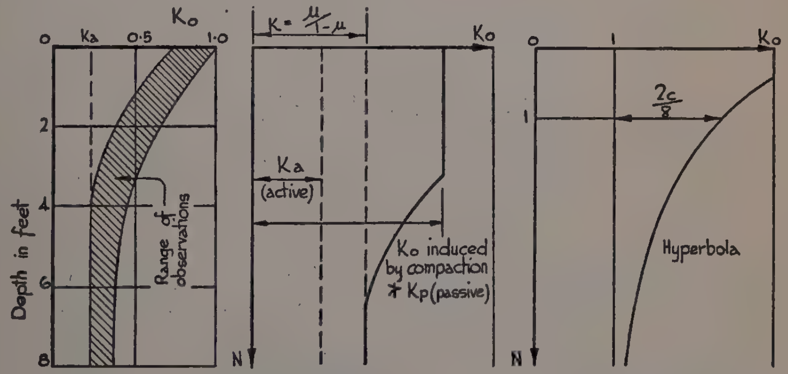
The strength of cohesive soils, both natural and remolded, depends on the moisture content, and it is as necessary in this method as in all others to test the soil at the greatest moisture content that is likely to exist during the lifetime of the pavement.

5. THE PRE-EXISTING STRESSES IN THE SOIL

Before any wheel load comes on to the pavement there exist, at any point in the soil, effective vertical stresses equal to the weight per unit of area of soil and pavement above the point in question, less any reduction due to buoyancy.

The effective horizontal stresses at any point can bear any relationship to the vertical stresses that is compatible with the envelope of failure, and the actual relationship in any case will depend on the history of the stratum.

In a cohesionless stratum the ratio of horizontal stress to vertical stress (K_o) can vary between $K_a = \dfrac{1 - \sin\phi}{1 + \sin\phi}$ and $K_p = \dfrac{1 + \sin\phi}{1 - \sin\phi}$, whereas in a frictionless stratum the horizontal stress can vary from the vertical stress by $\pm 2c$, and K_o can therefore vary between $1 \pm \dfrac{2c}{\gamma}\dfrac{1}{z}$, in which z is the depth below the surface of the stratum.



(a) Test on sand (b) Theoretical Ko Cohesionless soil compacted in layers (C) Theoretical maximum Ko Cohesive soil compacted in layers

FIG. 6.—PRE-EXISTING STRESSES IN THE SOIL

In the case of a frictionless stratum it might be useful to introduce a new term, the proportionate compaction, χ, defined by the equation:

$$\sigma_h = \sigma_v + \chi\, 2c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which σ_h and σ_v are the vertical and horizontal stresses in the unloaded stratum. Parameter χ will vary between $+1$ and -1.

It can be shown[21] that the actual value of K_o is likely to be a maximum at the surface in the case of strata in the bottom of cuts or in banks that have been built up by compaction in layers, whether the soil is cohesive or cohesionless. This condition seems to be confirmed by a test. Fig. 6 shows the actual values of K_o found in a test in a sand stratum compacted in 6-in. layers and the nature of the variation of K_o indicated by theory.

The importance of the value of K_o is immediately apparent when it is realized that, if K_o had its minimum value, the soil could carry no load beyond its own weight. Therefore, effective compaction will increase the carrying capacity of the subsoil.

6. BASIS OF COMPARISON WITH OTHER METHODS

In the attempt to emphasize any underlying common features in this and other methods, it is proposed to compare the results reported by Messrs. Westergaard and Glossop and Golder, and the results from CBR methods, with the proposed method. It will first be necessary to make an estimate of the linking magnitudes—that is, to establish a kind of "rate of exchange" between the measure of soil properties that form the basis of the various methods. These soil properties are:

(*a*) The strength of the foundation, represented by Eq. 1; and

(*b*) The elastic properties of the foundation material E_2 and μ_2, which are related to the CBR and the modulus of subgrade reaction, K_u.

In the case of an elastic half space the settlement of a rigid circular plate on the surface is given by the formula:

$$\delta = \frac{(1 - \mu^2{}_2)}{2 E_2} p\, a \qquad (5)$$

in which p is the average pressure on a plate of radius a. Hence,

$$K_u = \frac{p}{\delta} = \frac{2 E_2}{a(1 - \mu^2{}_2)} \qquad (6)$$

which is a relation between K_u and E.

Coefficient K_u is usually determined from the pressure required to cause a settlement of 0.05 in. on a plate of 15-in. radius. Consequently, it is found that

$$E_2 = 21\, K_u \qquad (7)$$

assuming $\mu_2 = \frac{1}{3}$.

Young's modulus E_2 can also be derived from the CBR by Eq. 7, if the appropriate deflection is chosen. A deflection of 0.00325 in. in the case of a disk with a 0.97-in. radius (such as is used in the CBR test) corresponds to the standard deflection of 0.05 in. found in the bearing test with a 30-in. plate. This deflection is very close to the origin of the load-deflection curve in the CBR test and the slope of the initial tangent may be taken as the appropriate value of p/δ. Assuming a parabola through the origin and the standard CBR pressures corresponding to deflections of 0.1 in. and 0.2 in. (Fig. 7), the slope of the tangent is 30,000 lb per in.[3] Hence,

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[21] "Bearing Capacity of Screw Piles and Screwcrete Cylinders," by G. Wilson, *Journal*, Inst. C. E., March, 1950, p. 4.

$$E_2 = 409\ \text{CBR} \qquad (8)$$

and

$$K_u = 19.5\ \text{CBR} \qquad (9)$$

The CBR could also be related to the strength of the foundation material by the formula proposed in 1947 by Karl Terzaghi,[22] Hon. M. ASCE, for the bearing capacity of a circular foundation, provided that an assumption is made about the deflection at failure,

$$q_{dr} = 1.3\,c\,N_c + \gamma\,D_f\,N_q + 0.6\,\gamma\,a\,N_\phi \qquad (10)$$

in which N_c, N_q, and N_ϕ are constants depending on ϕ, the angle of internal friction, and D_f is the depth of the footing below the surface of the ground. If failure occurs at a settlement of about 10% of the radius—that is, when an 100% CBR corresponds to a pressure of 1,000 lb per sq in., causing a settlement of 0.1 in.,

$$\text{CBR} = \frac{100 \times q_{dr}}{1{,}000} = \frac{q_d}{10} \qquad (11)$$

The CBR test is usually performed on clay soils without a surcharge so that the second term disappears from the right-hand side of Eq. 10 and the results obtained by this mode of calculation are in fair agreement with experimental values for this type of soil. Sands, on the other hand, are normally tested with a surcharge that could be allowed for by making the term γ/D_f equal to the pressure exerted on the sample by the surcharge. However, even if this is done, Eq. 11 gives results which are much smaller than those commonly obtained by experiment, so that it appears that the sample obtains additional restraint from the sides of the container and from the friction on the base of the surcharge. Consequently, Eq. 10 holds for clay soils but not for sands, and Eq. 9 is no longer applicable. However, a number of empirical relationships have been given connecting the CBR with K_u, one of which[23] the writers have adopted for cases when the CBR exceeds 10. Below this value, Eq. 9 is used and the resulting relationship between the CBR and K_u is shown at

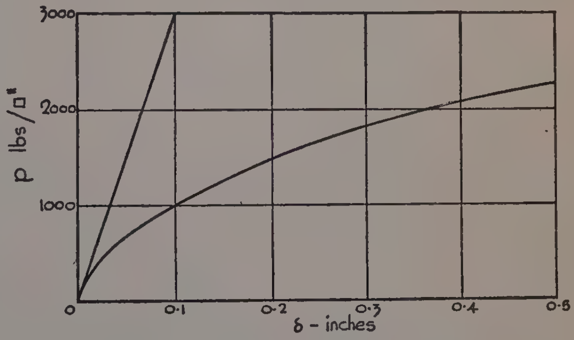

FIG. 7.—STANDARD TEST OF A 100% CBR MATERIAL; PLUNGER, 1.95 IN. IN DIAMETER

the top of Fig. 8. There seems to be no reason why Eq. 7 should not still be applicable.

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[22] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 2d Ed., 1947.

[23] "Handbook of Soils, Concrete and Bituminous Materials," H. M. Stationary Office, London, 1944, p. 191, Fig. C18.

Table 2, Cols. 1, 2, and 3, gives soil properties for four of the typical soils cited by Mr. Golder in 1946.[24] The data for the clays were obtained from Eqs. 8 to 11. As Eq. 10 is not applicable to sands, CBR values were chosen arbitrarily for loose and dense sands (types 3 and 4). The value for K_u (Col. 4), in pounds per cubic inch, was then derived empirically as outlined in the previous paragraph, and Eq. 7 was used to evaluate E_2 (Col. 6), in pounds per square inch. In Col. 3, Table 2, c is in pounds per square foot.

It was thought advisable to devise, as far as possible, a graduated scale connecting the various soil properties, rather than to use the range of values that Mr. Golder arbitrarily assigned on the basis of experience (a range with which the writers do not disagree). However, these relationships between the soil properties have been derived solely for comparing pavement thicknesses obtained by various methods, and it is not suggested that they should be applied in practice. The relationship between the modulus of elasticity and the modulus of subgrade reaction as determined by the plate bearing tests may be fairly reliable; and that between the modulus of elasticity and the CBR may be very far from the truth as the load-settlement curve is rarely parabolic as assumed. It may vary from a straight line to something like a fourth-power curve. The derivation of the CBR from the strength characteristics of the material is probably reasonably accurate in the case of clays.

TABLE 2.—LINKING OF SOIL PROPERTIES CONSIDERED IN VARIOUS METHODS

Soil type (1)	Description (2)	Shear strength (3)	K_u (4)	CBR (5)	E_2 (6)
1	Soft clay..........	$c = 400$	41	2.1	858
2	Stiff clay..........	$c = 2{,}000$	205	10.5	4,300
3	Loose sand..........	$\phi = 32°$	230	15	4,800
4	Dense sand..........	$\phi = 38°$	500	50	10,500

7. COMPARISON OF METHODS FOR DESIGNING CONCRETE PAVEMENTS

Fig. 8 shows the thickness of concrete pavement as required by Messrs. Westergaard, Glossop and Golder,[11] and Golder[6] in 1946, and the present method. It also shows the thickness of flexible pavement required by the CBR method for comparison with the suggestion made by Messrs. Henry and Grace.[16]

Scrutiny of the illustrations shows that the proposed method gives results generally proportional to those given by Mr. Westergaard's method. In the cases when failure occurs in the soil, a slight benefit arises if the soil can be so compacted that the "natural" horizontal stress is a maximum—that is, $\chi = 1$

The suggestion made by Messrs. Henry and Grace appears to be correc when the CBR is between 5 and 10. For smaller values a concrete pavemen could be less than half the thickness of a flexible pavement; but, if the CBR exceeds 10, the thickness of a concrete pavement would need to be more than half that of a flexible pavement. The CBR of the soil considered by Messrs. Henry and Grace was 3.

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[24] "The Relationship of Runway Thickness and Undercarriage Design to the Properties of the Subsoil, by H. Q. Golder, *Airport Paper No. 4*, Inst. C. E., 1946, p. 5, Table II.

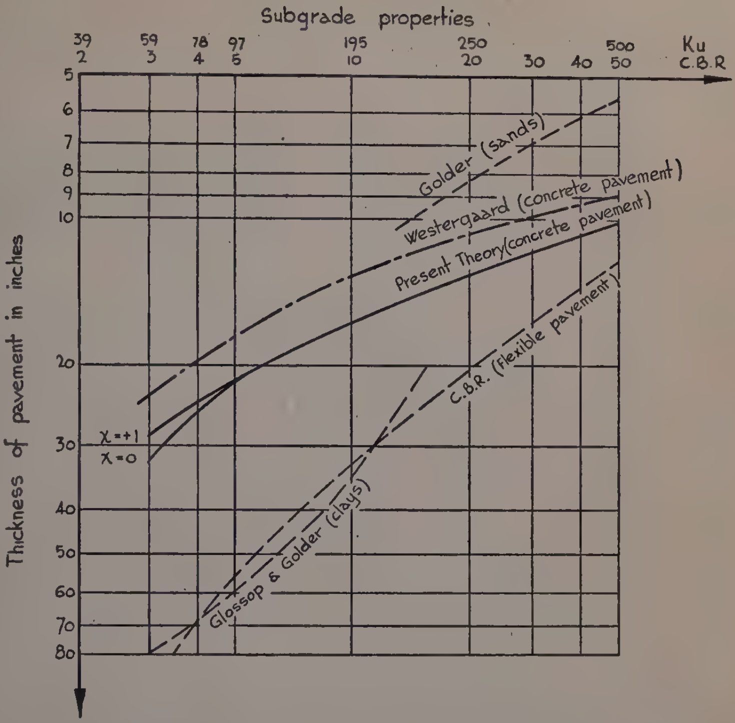

FIG. 8.—COMPARISON OF DESIGN METHODS (WHEEL LOAD, 100,000 LB; AND, IN POUNDS PER SQUARE INCH, THE TIRE PRESSURE IS 100 AND THE TENSILE STRENGTH OF CONCRETE IS 450)

The method proposed by Messrs. Glossop and Golder agrees closely with the CBR and it would therefore appear to be uneconomical for the design of concrete pavements. Mr. Golder's method,[6] on the other hand, appears to be unsafe, for the concrete would crack.

8. APPLICATION TO FLEXIBLE PAVEMENTS

The horizontal pressure at the base of an unloaded flexible pavement of compacted gravel or hard core is equal to $K'_o \gamma h$ (the coefficient K'_o being the ratio of horizontal to vertical stress in the unloaded pavement). Such a pavement may safely be loaded until the maximum tensile stress, $\overline{rr}$, at its base due to the surface load reaches this value. This stress may be calculated in the same way as the stress for concrete pavements, using suitable values for the ratio of the moduli of elasticity of the pavement and the subgrade materials. Since the CBR is proportional to the load required to produce a given compression in a sample of the soil, the ratio of the moduli may be taken approximately as the CBR of the subgrade material divided into 100 minus the CBR of a "perfect" pavement material. If this is done, loads known to be safe in practice would theoretically induce tensile stresses equal to many times the compressive stress $K'_o \gamma h$, even when very large values are taken for K'_o.

Apparently the pavement does not act elastically, and it is suggested that it behaves approximately as follows: Fig. 9 shows the effect of variations in the ratio of the elastic moduli E_1/E_2, on the stress $\overline{rr}$ and shows that a reduction in E_1/E_2 causes a reduction in $\overline{rr}$. If E_1/E_2 is initially based on the CBR of the subgrade, $\overline{rr}$ increases rapidly with an increasing load and its safe value, $K'_o \gamma h$, is reached very quickly. The pavement must then yield, which is roughly equivalent to a decrease occurring in E_1, its elastic modulus; and this action results in a decrease in $\overline{rr}$, which may continue until $\overline{rr} = K'_o \gamma h$ when equilibrium is reached—that is,

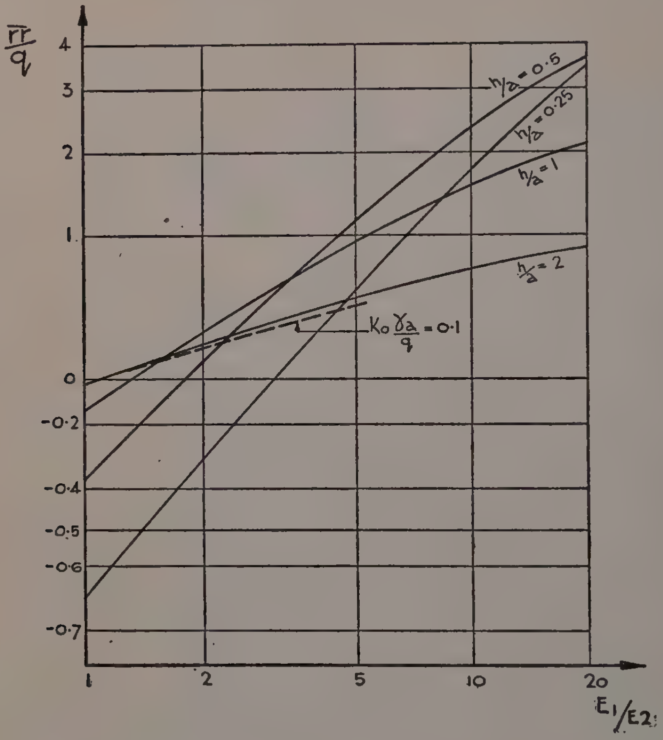

FIG. 9

$$\frac{\overline{rr}}{q} = K'_o \frac{\gamma a}{q} \frac{h}{a} \quad \text{(12)}$$

For a given value of $K'_o \frac{\gamma a}{q}$, Eq. 12 defines a line in Fig. 9 intersecting the curves corresponding to various values of h/a so as to give the apparent value of E_1/E_2 when $\overline{rr}$ is equal to $K'_o\,\gamma\,h$ in any particular pavement. A typical intersecting curve is given for $K'_o\,\dfrac{\gamma\,a}{q} = 0.1$, showing that if, for instance, $h/a = \frac{1}{4}$ the pavement behaves as if $E_1/E_2 = 4.5$. It seems reasonable to assume that the stresses in the subgrade are approximately equal to those set up when E_1/E_2 is equal to 4.5, and they may be calculated from the Burmister theory, after the manner suggested by Mr. Fox. Consequently, if $K'_o\,\dfrac{\gamma\,a}{q}$ is given, the maximum shear stress in the subgrade may be calculated for each value of h/a. Equating this quantity to the cohesion c, a curve may be drawn giving safe values of a/h as a function of c/q for purely cohesive subgrades.

Fig. 10 shows two such curves for $K'_o\,\dfrac{\gamma\,a}{q} = 0$ and 0.1, assuming $K_o = 1$ in the subgrade. As these values represent the upper and lower limits likely to be encountered, variations in this parameter have little effect on the result. The third curve in Fig. 10 was obtained by direct application of the Boussinesq theory, assuming the pavement and subgrade, jointly, to constitute a semi-infinite, homogeneous elastic solid, and equating the maximum shear stress in the subgrade to its cohesion. The similarity between the curves suggests that flexible pavements behave very much according to the Boussinesq theory, an hypothesis stated in 1949 by R. de l'Hortet,[25] who showed the close relation between results obtained by the CBR method and those obtained by the Boussinesq theory.

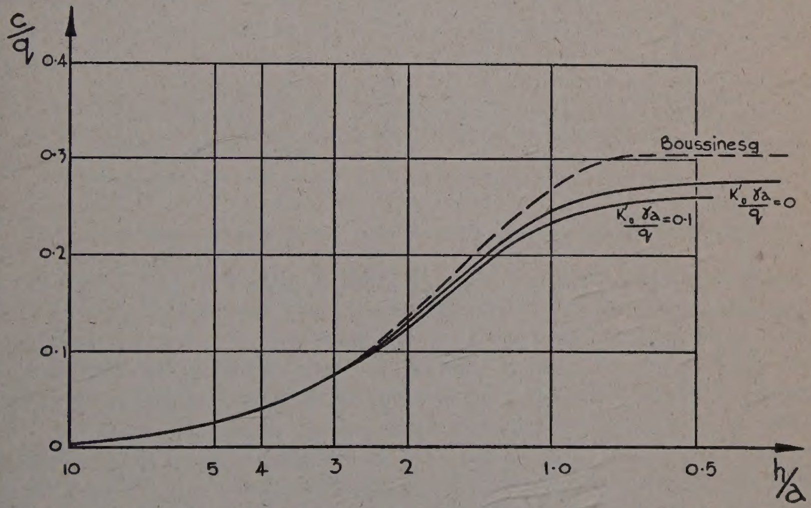

FIG. 10.—RELATION BETWEEN THE THICKNESS REQUIRED FOR A FLEXIBLE PAVEMENT ON A FRICTIONLESS SUBGRADE, AND THE COHESION (COEFFICIENT OF EARTH PRESSURE AT REST IN THE SUBGRADE, $K'_o = 1$)

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[25] "Runway Strength Correlation Between the Distribution of Stresses in the Soil Derived from: (1) The C.B.R. Method of the Determination of Runway Thickness; (2) the Theory of Boussinesq," by R. de l'Hortet, *Proceedings*, 2d International Conference on Soil Mechanics, Rotterdam, 1948, Vol. V, p. 185.

It is difficult to obtain a direct comparison between the thickness derived for a particular case by the CBR method and the present method. For exact equivalence the two methods should give the same thickness for any frictionless soil and loading. Thus, if for any soil the least thickness required by the CBR is used to compute c from Fig. 10, the results should be independent of the particular wheel load and tire pressure chosen. Taking thicknesses from a CBR chart,[26] the relationship in Fig. 11 was obtained. Three curves are shown working with wheel loads of 4,000 lb, 40,000 lb, and 70,000 lb, the discrepancy suggesting that the CBR has a slightly higher safety factor for the smaller wheel loads. The results for the cohesion are considerably affected by the tire pressure, which is not allowed for very closely by the CBR method. A pressure of 60 lb per sq in. was assumed, but a slight increase or decrease in the case of the lightly loaded or heavily loaded wheels, respectively, would bring the curves more closely together. The broken line in Fig. 11 shows the relationship between the CBR and the cohesion of the subgrade which was obtained by an independent method in Section 6. It will be seen that the two methods give very similar results.

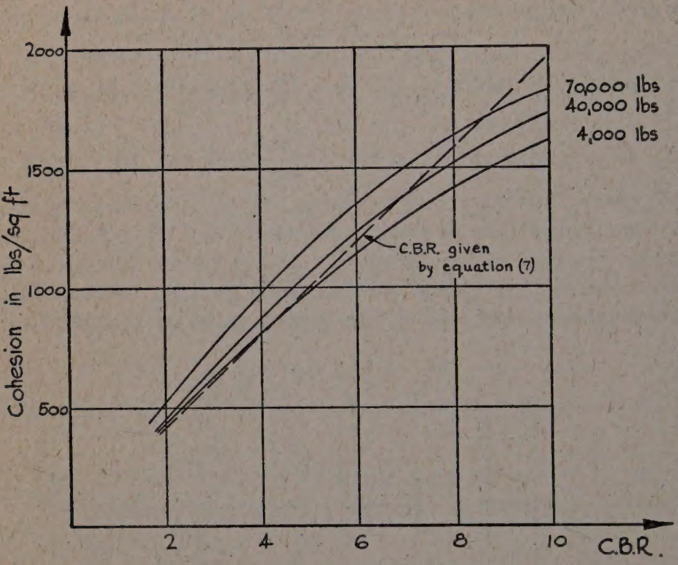

Fig. 11.—Relation Between the CBR and the Cohesion of Subgrade Materials

The method may be applied to frictional subgrade materials by assuming the same values for E_1/E_2 and by taking Eq. 1 as the criterion for subgrade failure.

In the case of thick flexible pavements (of thickness h approximately greater than the radius a), the pavement material may also fail in a manner similar to the failure of material under a simple foundation, a possibility that can be allowed for by the usual methods.

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[26] "Handbook of Soils, Concrete and Bituminous Materials," H. M. Stationery Office, London, 1944, p. 191.

9. LIMITATIONS OF PROPOSED METHOD

The obvious limitations of the proposed method are that it necessarily relates only to a load on the surface of the upper layer of a distinct two layered system. Edge and corner conditions that control the design of concrete surface pavements, and the gradual merging of flexible surfaces into subgrade, via base and subbase, cannot be provided for in the theory.

Edge and corner conditions, which are considered only in Mr. Westergaard's method, could be investigated by the methods of three-dimensional photoelasticity. Loads could be applied to large blocks of bakelite through the edge and corners of metal sheets of varying thickness and varying elastic moduli. Direct comparison with Mr. Westergaard's method for the "interior" of slabs will be useful, in any case.

However, the method is directly applicable to pre-stressed concrete pavements and to the "schipol sandwich" type of construction.[27] Both these types of pavement seem destined for wide adoption.

A limitation that is less immediately apparent, but which is, none the less, real, is that the modulus of elasticity of soils depends on the state of stress and that, consequently, the distribution of stress from a superimposed load depends on the pre-existing state of stress in the system. Against this limitation, as was so brilliantly stated by A. Caquot[28] in 1948, is the fact that it seems to be possible to "educate" soils, by repetition of loading, to behave more as elastic materials.

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[27] "Soil Investigations and Runway Construction at Schipol Airport," by J. W. Clerx and L. J. H. Weinberg, *Proceedings*, 2d International Conference on Soil Mechanics, Rotterdam, 1948, Vol. IV. p. 197.

[28] "Foundation Pressure and Settlements of Buildings on Footings and Rafts: General Report," by A. Caquot, *ibid.*, Vol. VI, Section VI, p. 112.